



KASR TARTIBLI BIR O'LCHOVLI DIFFERENSIAL TENGLAMALAR UCHUN TESKARI MASALA

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Annotatsiya: Ushbu tezisda kasr tartibli bir o'lchovli differensial tenglamalar uchun teskari masalalarni o'rganish masalasi qaraladi. Kasr tartibli differensial tenglamalar an'anaviy differensial tenglamalardan farq qilgan holda, ularning tartibini butun emas, ratsional son yoki kasr ko'rinishda bo'ladi. Tadqiqotning asosiy maqsadi kasr tartibli differensial tenglamada noma'lum koeffitsiyent yoki manba funksiyasini qo'shimcha o'lchov ma'lumotlari asosida aniqlashdan iborat. Teskari masalaning yechilish shartlari, yechimning yagonaligi va barqarorligi masalalarini o'rganish uchun matematik modellashtirish hamda funksional analiz usullaridan foydalanish ko'zda tutiladi.

Kalit so'zlar: Kasr tartibli differensial tenglamalar, yechim usullari, tatbiq sohalari, matematik modellashtirish, fizik jarayonlar, teskari masala, Riman–Liuvill hosilasi,

Kasr tartibli hisoblash ko'plab amaliy masalalarni matematik modellashtirishda muhim rol o'ynaydi. Masalan, ishda chiziqli bo'lmagan maksimalarga ega Hilfer tipidagi kasr differensial tenglama uchun boshlang'ich qiymat masalasining yechilishi tadqiq etilgan. Analitik usul yordamida Hilfer kasr operatori qatnashgan kuchsiz chiziqli bo'lmagan aralash turdagi qisman differensial tenglama uchun chegaraviy qiymat masalasining yagona yechilishi o'rganilgan. Hilfer kasr operatoriga ega aralash turdagi to'rtinchi tartibli differensial tenglama uchun nolokal masala yechilishi tadqiq etilgan.

Ushbu tezisda Barenblatt–Jeltov–Kochina operatorining kvadrati, degenerat yadro hamda berilgan kesmaning oxirgi nuqtasida qayta aniqlanadigan funksiya ishtirok etuvchi bir jinsli integro-differensial tenglama



uchun teskari chegara qiymat masalasi o'rganiladi. Mazkur masalada yechimining mavjudligi va yagonaligi o'rganiladi.

Berilgan kesmaning oxirgi nuqtasida qayta aniqlanuvchi funksiyaga ega bo'lgan chiziqli qismaniy differensial tenglama uchun teskari chegara qiymat masalasi ko'rib chiqiladi. Mazkur teskari chegara qiymat masalasining yechimining mavjudligi va yagonaligi masalalari tadqiq etiladi.

$$\Omega = \{(t, x) | 0 < t < T, 0 < x < l\}$$

sohada quyidagi differensial tenglamalar ko'rib chiqiladi

$$\left[CD_{0t}^{\alpha} - {}_c D_{0t}^{\alpha} \frac{\partial^2}{\partial x^2} - t^{\beta} \frac{\partial^2}{\partial x^2} \right]^2 u(t, x) = a(t)u(x, y) + f(t, x) \quad (1)$$

chegaravviy shartlar bilan

$$u(0, x) = \varphi_1(x), u(T, x) = \varphi_2(x), 0 \leq x \leq l, \quad (2)$$

o'rganiladi

bu yerda β , T va l musbat haqiqiy sonlar berilgan, $\varphi_{\kappa}(x) \in C(\Omega_l)$, $\varphi_{\kappa}(0) = \varphi_{\kappa}(l) = \varphi_{\kappa}''(0) = \varphi_{\kappa}''(l) = 0$, $\kappa = 1, 2$.

Riman-Liouville integral operatori $a(t) \in C(\Omega_T)$ $f(t, x) \in C(\Omega_T \times \Omega_l)$, $\varphi_{\kappa}(x)$ ($\kappa = 1, 2$) berilgan funksiya, $\Omega_T \equiv [0; T]$, $\Omega_l \equiv [0; l]$.

1-masala. Biz (1) xususiy differensial tenglamani hamda boshlang'ich va (2) yakuniy shartlarni, shuningdek Dirixle shartlarini qanoatlantiruvchi $u(t, x) \in C(\overline{\Omega})$ funksiyani topish talab etiladi.

$$u(t, 0) = u(t, l) = u_{xx}(t, 0) = u_{xx}(t, l) = 0, \quad (3)$$

Bu yerda

$$\overline{\Omega} = \{(t, x) | 0 \leq t \leq T, 0 \leq x \leq l\}$$



Faraz qilaylik

$$u(t, x) = \sum_{n=1}^{\infty} u_n(t) \cdot \vartheta_n(x) \quad (4)$$

va

$$f(t, x) = \sum_{n=1}^{\infty} f_n(t) \cdot \vartheta_n(x), (t, x) \in \Omega, \quad (5)$$

bu yerda

$$u_n(t) = \int_0^l u(t, y) \vartheta_n(y) dy, f_n(t) = \int_0^l f(t, y) \vartheta_n(y) dy, \quad (6)$$

$$\vartheta_n(x) = \sqrt{\frac{2}{l}} \sin \lambda_n x, \lambda_n = \frac{n\pi}{l}, n = 1, 2, \dots \quad (7)$$

E'tibor beramizki, (7) dagi $\{\vartheta_n(x)\}_{n=1}^{\infty}$ funksiyalar sistemasi $L_2(\Omega_l)$ fazoda ortonormal funksiyalarning to'liq sistemasini hosil qiladi va chegara shartlarini qanoatlantiradi:

$\vartheta_n(0) = \vartheta_n(l) = \vartheta_n''(0) = \vartheta_n''(l) = 0$. Binobarin, Furye qatori (4) bilan ifodalangan funksiyalar (3) shartlarini formal revishda qanoatlantiradi.

Biz (1) tenglamani quyidagi shaklda qayta yozamiz:

$$\left[(CD_{0t}^{\alpha})^2 - 2(CD_{0t}^{\alpha})^2 \frac{\partial^2}{\partial x^2} + (CD_{0t}^{\alpha})^2 \frac{\partial^4}{\partial x^4} - 2t^{\beta} {}_c D_{0t}^{\alpha} \frac{\partial^2}{\partial x^2} + \right. \\ \left. + 2t^{\beta} {}_c D_{0t}^{\alpha} \frac{\partial^4}{\partial x^4} + t^{2\beta} \frac{\partial^4}{\partial x^4} \right] u(t, x) = a(t)u(x, y) + f(t, x).$$

(4) va (5) Furye qatorlarini (1) differensial tenglamaga qo'yib, quyidagi natijani olamiz.



$$\begin{aligned} & \sum_{n=1}^{\infty} (CD_{0t}^{\alpha})^2 u_n(t) \vartheta_n(x) - 2 \sum_{n=1}^{\infty} (CD_{0t}^{\alpha})^2 \vartheta_n''(x) + \sum_{n=1}^{\infty} (CD_{0t}^{\alpha})^2 \vartheta_n^{(IV)}(x) - \\ & - 2 \sum_{n=1}^{\infty} t^{\beta} {}_C D_{0t}^{\alpha} u_n(t) \vartheta_n''(x) + 2 \sum_{n=1}^{\infty} t^{\beta} {}_C D_{0t}^{\alpha} u_n(t) \vartheta_n^{(IV)}(x) \\ & + \sum_{n=1}^{\infty} t^{2\beta} u_n(t) \vartheta_n^{(IV)}(x) = \\ & = a(t) \sum_{n=1}^{\infty} u_n(t) \vartheta_n(x) + \sum_{n=1}^{\infty} f_n(t) \vartheta_n(x). \end{aligned}$$

(7) dagi funksiyaning xossalarini hisobga olgan holda, oxirgi munosabatdan quyidagi natijalarni hosil qilamiz:

$$\begin{aligned} & \sum_{n=1}^{\infty} [1 + 2\lambda_n^2 + \lambda_n^4] (CD_{0t}^{\alpha})^2 u_n(t) \vartheta_n(x) + \sum_{n=1}^{\infty} 2t^{\beta} \lambda_n^2 [1 + \lambda_n^2] {}_C D_{0t}^{\alpha} u_n(t) \vartheta_n(x) \\ & + \sum_{n=1}^{\infty} t^{2\beta} u_n(t) \lambda_n^4 \vartheta_n(x) \\ & = a(t) \sum_{n=1}^{\infty} u_n(t) \vartheta_n(x) + \sum_{n=1}^{\infty} \int_0^l f(t, y) \vartheta_n(y) dy \cdot \vartheta_n(x). \end{aligned}$$

Shunday qilib, har bir hadni (6) Furrye koeffitsientlariga nisbatan $\vartheta_m(x)$ ga skalyar ravishda ko'paytirib, biz Hilfer operatori bilan sanoqli differensial tenglamalar sistemasiga kelimiz.

Silliqlik sharti 1. Aytaulik $\varphi_{\kappa}(x) \in C^2(\Omega_l)$ bo'lsin.

1-teorema. Agar

$$\rho_1 = a_0 \chi_2 M_3 (1 + M_4) \frac{T^{\alpha+1}}{\alpha + 1} < 1.$$



Tengsizlik o'rinli bo'lsa, chiziqli integral tenglamalarning sanoqli sistemasini $B_2(\Omega_T)$ fazoda (yagona) yechimga ega ekanligi isbotlanadi.

Kasr tartibli differensial tenglamalarni yechish an'anaviy differensial tenglamalarga qaraganda murakkabroq va ko'pincha analitik yechim olish qiyin bo'ladi. Shu sababli zamonaviy metodlar va usullar qo'llaniladi. Yechim olishning asosiy yo'nalishlaridan biri quyidagi hisonlanadi:

- Siquvchi akslantirish metodi: Ushbu usul yechimning mavjudligi va yagonaligini isbotlash uchun qo'llaniladi. Unga ko'ra operatorning yagona qo'zg'almas nuqtasi mavjud va bu nuqta dastlabki kasr tartibli differensial tenglamaning yagona yechimi hisoblanadi[2].

Kasr tartibli differensial tenglamalar nafaqat nazariy izlanishlarda, balki ko'plab amaliy sohalarda ham keng qo'llaniladi. Ularning xotira va esdan chiqarish effektlarini hisobga olish qobiliyati, tabiiy jarayonlarda mavjud noan'anaviy dinamikani yaxshiroq tavsiflash imkonini beradi. Misol uchun:

- Fizika: Kasr tartibli differensial tenglamalar to'qimalar va materiallarning mexanik xatti-harakatlarini tushuntirishda, ayniqsa materiallarning deformatsiyasini o'rganishda qo'llaniladi.

Xulosa

Kasr tartibli bir o'lchovli differensial tenglamalar uchun teskari masalalarni o'rganish zamonaviy matematik analizning muhim yo'nalishlaridan biridir. Bunday masalalarda kasr tartibli hosilaning integral xususiyati hamda noma'lum parametrlarning yechimga ta'siri asosiy tadqiqot obyektni tashkil etadi.

Mazkur tadqiqotda Kaputo kasr tartibli hosilasi qatnashgan differensial tenglama uchun noma'lum koeffitsiyentni qo'shimcha o'lchov sharti yordamida aniqlash masalasi shakllantirildi. Teskari masalaning mavjudligi, yagonaligi va barqarorligi bilan bog'liq masalalarni integral tenglamalar va funksional analiz usullari yordamida o'rganish rejalashtirildi. Kelgusida



maxsus shartlar ostida aniq teoremlarni isbotlash va sonli tajribalar o'tkazish tadqiqotning asosiy yo'nalishlarini tashkil etadi.

Foydalanilgan adabiyotlar ro'yxati

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