

**IRRATSIONAL FUNKSIYALAR BILAN BOG'LIQ
EKSTREMAL MASALALARNI VA TENGLAMALARNI
GEOMETRIK USULLARDA YECHISH METODIKASI.**

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Annotatsiya. Ushbu maqolada irratsional funksiyalar bilan bog'liq ayrim masalalar hamda tenglamalarni yechishni geometrik yechish usullari haqida so'z borgan.

Kalit so'zlar: Tenglama, ekstremal, yechim, funksiya, irratsional, masala.

Абстрактный. В данной статье обсуждаются некоторые вопросы, связанные с иррациональными функциями и геометрическими решениями для решения уравнений.

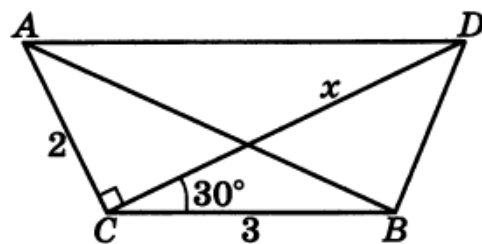
Ключевые слова: Уравнение, экстремум, решение, функция, иррациональное, задача.

Abstract. This article deals with some issues related to irrational functions and geometric solutions for solving equations.

Key words: Equation, extreme, solution, function, irrational, problem.

1. $f(x) = \sqrt{x^4 + 4} + \sqrt{x^2 - 3x\sqrt{3} + 9}$ funksiyaning eng kichik qiymatini toping.

Yechish. 1–chizmada $\triangle ACD$ ($AC = 2, CD = x, \angle ACD = 90^\circ$) va $\triangle BCD$ ($BC = 3, CD = x, \angle BCD = 30^\circ$)



1-chizma

$\triangle ACD$ dan Pifagor teoremasiga ko‘ra $AD = \sqrt{x^2 + 4}$

$\triangle BCD$ dan kosinuslar teoremasiga ko‘ra $DB = \sqrt{x^2 + 9 - 3x\sqrt{3}}$

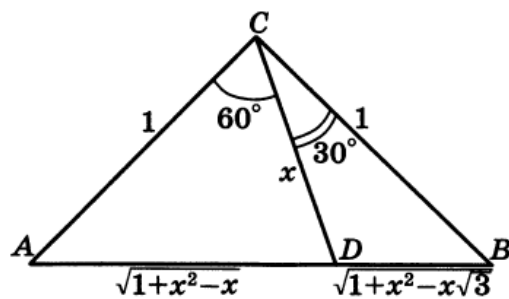
$$\min f(x) = \min(AD + DB) = AB$$

$\triangle ABC$ uchburchakdan kosinuslar teoremasiga ko‘ra

$$AB = \sqrt{2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cos 120^\circ} = \sqrt{19}.$$

2. x ning qanday qiymatlarida $f(x) = \sqrt{1+x^2} - x + \sqrt{1+x^2} - x\sqrt{3}$ funksiya o‘zining eng kichik qiymatiga erishadi?

Yechish. 2 –chizmadan $\triangle ABC$, $\triangle ACD$ va $\triangle BCD$ uchburchaklar yuzini topamiz.



2-chizma

$$S_{\triangle ABC} = \frac{1}{2}.$$

$$S_{\triangle ACD} = \frac{1}{2} \cdot 1 \cdot x \cdot \sin 60^\circ = \frac{x\sqrt{3}}{4}.$$

$$S_{\triangle BCD} = \frac{1}{2} \cdot 1 \cdot x \cdot \sin 30^\circ = \frac{x}{4}.$$

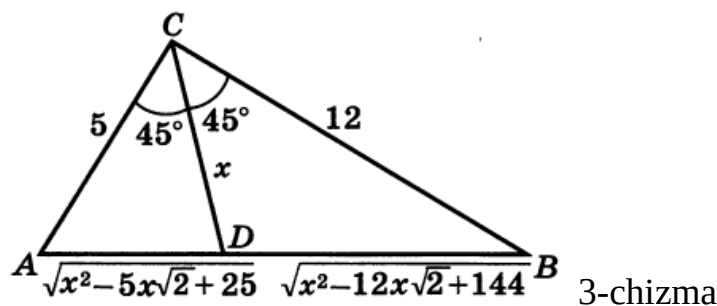
$$\text{Demak, } \frac{1}{2} = \frac{x\sqrt{3}}{4} + \frac{x}{4}; \quad x = \sqrt{3} - 1.$$

Javob: $\sqrt{3} - 1$

$$3. \sqrt{x^2 - 5x\sqrt{2} + 25} + \sqrt{x^2 - 12x\sqrt{2} + 144} = 13 \text{ tenglamani yeching.}$$

Yechish. Bu tenglamaning an'anaviy usulda yechish uchun har tomoni ildizlar yo'qolmaguncha kvadratga ko'tariladi. Tenglamani geometrik usulda yechish qisqa muddatda amalga oshiriladi. 3–chizmada $\triangle ABC$ dan Pifagor teoremasiga ko'ra $AB = 13$.

$D \in AB$, chunki $\min f(x) = \min(AD + DB) = AB$.



Bunda $f(x) = \sqrt{x^2 - 5x\sqrt{2} + 25} + \sqrt{x^2 - 12x\sqrt{2} + 144}$ x ning qiymatini topish uchun 2-misoldagidek ish yuritamiz. Agar tenglama ildizlarga ega bo'lsa, ular musbat bo'lishni aniqlaymiz. ($x \leq 0$ da tenglama chap qismining qiymati 12 dan kichik emas).

$$S_{\triangle ABC} = \frac{1}{2} \cdot 5 \cdot 12 = 30.$$

$$S_{\triangle ACD} = \frac{1}{2} \cdot 5 \cdot x \cdot \sin 45^\circ = \frac{5x\sqrt{2}}{4}.$$

$$S_{\triangle BCD} = \frac{1}{2} \cdot 12 \cdot x \cdot \sin 45^\circ = 3x\sqrt{2}.$$

Demak, $30 = \frac{5x\sqrt{2}}{4} + 3x\sqrt{2}; \quad x = \frac{60\sqrt{2}}{17}.$

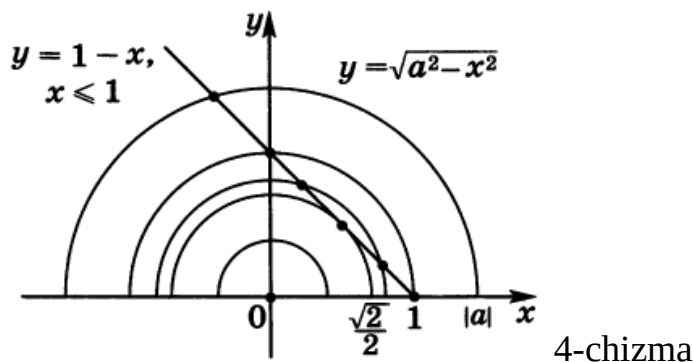
Javob:

$\frac{60\sqrt{2}}{17}.$

4. a parametrning har bir qiymatida $\sqrt{a^2 - tg^2 z} = 1 - tgz$ tenglamada tgz nechta har xil qiymatlar qabul qiladi.

Yechish. Bu oddiy tenglama emas. Uni an'anaviy usulda yechish oson emas, agar tgz ni x bilan almashtirsak, uni geometrik usulda yechish oson bo'ladi.

4-chizmada $y = \sqrt{a^2 - x^2}$ (markazi koordinatalar boshida, radiusi $|a|$ bo'lgan yuqori yarim tekislikda joylashgan yarim aylana) va $y = 1 - x$ ($x \leq 1$) (boshi $(1;0)$ nuqtada $(0,1)$ nuqtadan o'tuvchi nur) funksiyalar grafiklari tasvirlangan.



Demak, $\sqrt{a^2 - x^2} = 1 - x$

$|a| < \frac{\sqrt{2}}{2}$ da ildiz yo'q.

$|a| = \frac{\sqrt{2}}{2}$ va $|a| > 1$ da 1 ta ildizga ega.

$$\frac{\sqrt{2}}{2} < |a| \leq 1 \text{ da } 2 \text{ ta ildizga ega.}$$

Demak, tgz :

$|a| = \frac{\sqrt{2}}{2}$ da bitta qiymat qabul qiladi. ($-\frac{\sqrt{2}}{2}$ da bitta son bo'lsa, $\frac{\sqrt{2}}{2}$ da boshqa sonidir).

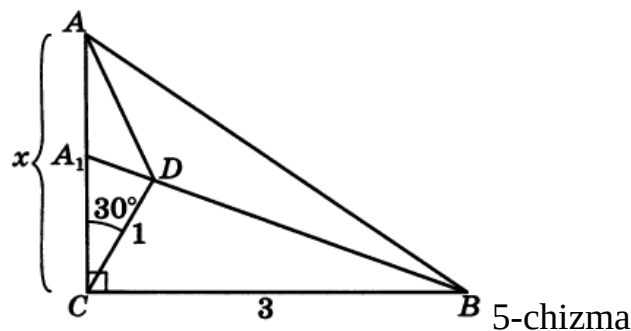
$|a| > 1$ da bitta qiymat qabul qiladi.

$$\frac{\sqrt{2}}{2} < |a| \leq 1 \text{ da } 2 \text{ ta har xil qiymat qabul qiladi.}$$

5. $f(x) = \sqrt{x^2 + 9} - \sqrt{x^2 - x\sqrt{3} + 1}$ funksiyaning eng katta qiymatini toping.

Yechish. ABC uchburchakni qaraymiz.

Bunda $\angle ACB = 90^\circ$, $\angle ACD = 30^\circ$, $AC = x$, $BC = 3$, $CD = 1$ va D nuqta ABC uchburchak ichida yotadi. (5-chizma)



ABC uchburchakdan Pifagor teoremasiga ko'ra $AB = \sqrt{x^2 + 9}$.

$\triangle ACD$ dan kosinuslar teoremasiga ko'ra $AD = \sqrt{x^2 - x\sqrt{3} + 1}$.

$$\max f(x) = \max(AB - AD) = A_1B - A_1D = DB,$$

bunda $A_1 \in AC$ (agar $D \in AB$).

$\triangle BCD$ dan kosinuslar teoremasiga ko'ra

$$DB = \sqrt{1^2 + 3^2 - 2 \cdot 1 \cdot 3 \cos 60^\circ} = \sqrt{7}.$$

Javob: $\sqrt{7}$.

6. $f(x) = x\sqrt{2} + \sqrt{2x^2 + 2x + 1} + \sqrt{2x^2 - 14x + 25} + \sqrt{2x^2 - 26x + 89}$
funksiyaning eng kichik qiymatini toping.

Yechish. $\vec{a}(x; x)$, $\vec{b}(4 - x; 3 - x)$, $\vec{c}(x + 1; x)$, $\vec{d}(5 - x; 8 - x)$ vektorlar va ularning modullarini qaraymiz.

$$|\vec{a}| = x\sqrt{2}.$$

$$|\vec{b}| = \sqrt{2x^2 - 14x + 25}.$$

$$|\vec{c}| = \sqrt{2x^2 + 2x + 1}.$$

$$|\vec{d}| = \sqrt{2x^2 - 26x + 89}.$$

$$f(x) = |\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}|.$$

$$|\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}| \geq |\vec{a} + \vec{b} + \vec{c} + \vec{d}| \text{ bo'lgani uchun}$$

$$\min f(x) = |\vec{a} + \vec{b} + \vec{c} + \vec{d}|.$$

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\vec{10}; \vec{11})$$

$$|\vec{a} + \vec{b} + \vec{c} + \vec{d}| = \sqrt{221}.$$

Demak, $\min f(x) = \sqrt{221}.$

Javob: $\sqrt{221}$.

7. $f(x) = \sqrt{2x^2 - 4x + 2} + \sqrt{2x^2} + \sqrt{2x^2 - 2x + 2}$ funksiyaning eng kichik qiymatini toping.

Yechish. 4 ta $O(0;0)$, $A(1;1)$, $B\left(\frac{1-\sqrt{3}}{2}; \frac{1+\sqrt{3}}{2}\right)$, $M(x;x)$ va OM , AM va

BM masofalarni qaraymiz.

$$OM = \sqrt{2x^2}, \quad AM = \sqrt{2x^2 - 4x + 2}, \quad BM = \sqrt{2x^2 - 2x + 2}.$$

$$f(x) = OM + AM + BM.$$

ABO uchburchakning tomonlarini hisoblash qiyin emas.

$AO = BO = AB = \sqrt{2}$. Demak, $\triangle ABO$ teng tomonli

$$\min(AM + OM + BM) = 3R,$$

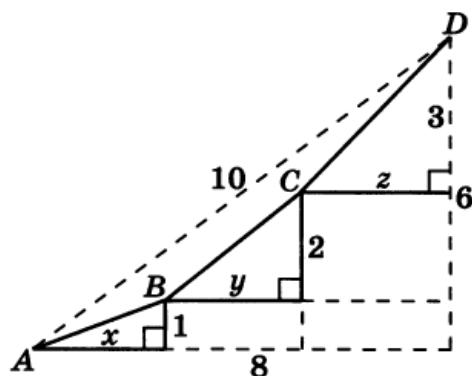
R -teng tomonli uchburchakka tashqi chizilgan aylana radiusi.

$$\sqrt{2} = R\sqrt{3} \text{ bo'lgani uchun } \min f(x) = \sqrt{6}.$$

Javob: $\sqrt{6}$.

8. $f(x; y; z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 4} + \sqrt{z^2 + 9}$ va $x + y + z = 8$ bo'lsa, $f(x; y; z)$ funksiyaning kichik qiymatini toping.

Yechish.



6-chizma

6-chizmaga ko'ra $ABCD$ egri chiziqli uzunligi 10 dan kichik emas, u holda $x + y + z = 8$ da $f(x; y; z) = 10$.

10.

FOYDALANILGAN ADABIYOTLAR RO'YHATI.

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