

# IRRATSIONAL FUNKSIYALAR BILAN BOG'LIQ EKSTREMAL MASALALARNI VA TENGLAMALARNI GEOMETRIK USULLARDA YECHISH METODIKASI.

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*Annotatsiya. Ushbu maqolada irratsional funksiyalar bilan bog'liq ayrim masalalar hamda tenglamalarni yechishni geometrik yechish usullari haqida so'z borgan.*

*Kalit so'zlar: Tenglama, ekstremal, yechim, funksiya, irratsional, masala.*

*Абстрактный. В данной статье обсуждаются некоторые вопросы, связанные с иррациональными функциями и геометрическими решениями для решения уравнений.*

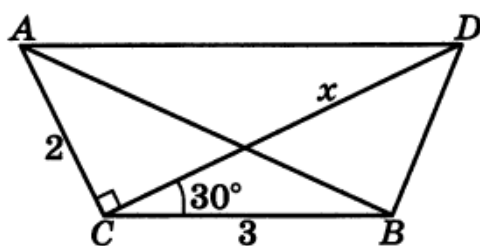
*Ключевые слова: Уравнение, экстремум, решение, функция, иррациональное, задача.*

**Abstract.** This article deals with some issues related to irrational functions and geometric solutions for solving equations.

**Key words:** Equation, extreme, solution, function, irrational, problem.

1.  $f(x) = \sqrt{x^4 + 4} + \sqrt{x^2 - 3x\sqrt{3} + 9}$  funksiyaning eng kichik qiymatini toping.

Yechish. 1-chizmada  $\triangle ACD$  ( $AC = 2, CD = x, \angle ACD = 90^\circ$ ) va  $\triangle BCD$  ( $BC = 3, CD = x, \angle BCD = 30^\circ$ )



1-chizma

$\triangle ACD$  dan Pifagor teoremasiga ko'ra  $AD = \sqrt{x^2 + 4}$

$\triangle BCD$  dan kosinuslar teoremasiga ko'ra  $DB = \sqrt{x^2 + 9 - 3x\sqrt{3}}$

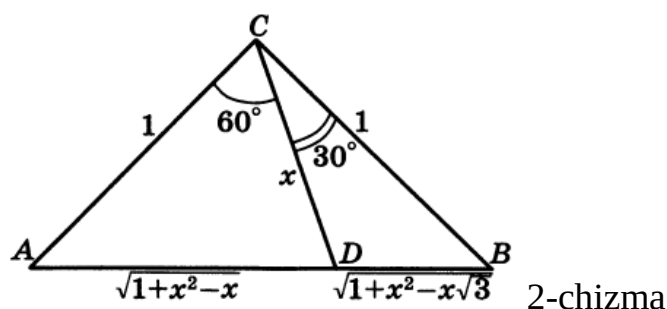
$$\min f(x) = \min(AD + DB) = AB$$

$ABC$  uchburchakdan kosinuslar teoremasiga ko'ra

$$AB = \sqrt{2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cos 120^\circ} = \sqrt{19}.$$

2.  $x$  ning qanday qiymatlarida  $f(x) = \sqrt{1+x^2-x} + \sqrt{1+x^2-x\sqrt{3}}$  funksiya o'zining eng kichik qiymatiga erishadi?

Yechish. 2 -chizmadan  $ABC$ ,  $ACD$  va  $BCD$  uchburchaklar yuzini topamiz.



$$S_{\triangle ABC} = \frac{1}{2}.$$

$$S_{\triangle ACD} = \frac{1}{2} \cdot 1 \cdot x \cdot \sin 60^\circ = \frac{x\sqrt{3}}{4}.$$

$$S_{\triangle BCD} = \frac{1}{2} \cdot 1 \cdot x \cdot \sin 30^\circ = \frac{x}{4}.$$

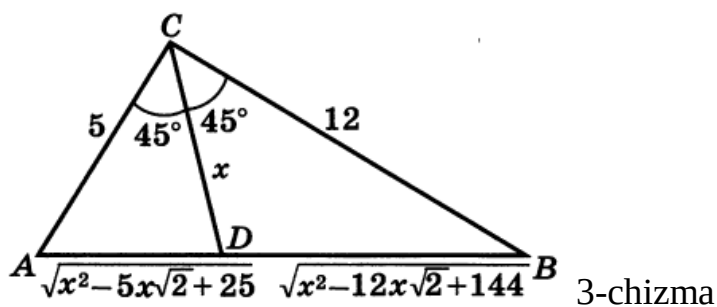
$$\text{Demak, } \frac{1}{2} = \frac{x\sqrt{3}}{4} + \frac{x}{4}; \quad x = \sqrt{3} - 1.$$

Javob:  $\sqrt{3} - 1$ .

$$3. \sqrt{x^2 - 5x\sqrt{2} + 25} + \sqrt{x^2 - 12x\sqrt{2} + 144} = 13 \text{ tenglamani yeching.}$$

Yechish. Bu tenglamaning an'anaviy usulda yechish uchun har tomoni ildizlar yo'qolmaguncha kvadratga ko'tariladi. Tenglamani geometrik usulda yechish qisqa muddatda amalga oshiriladi. 3-chizmada  $\triangle ABC$  dan Pifagor teoremasiga ko'ra  $AB = 13$ .

$D \in AB$ , chunki  $\min f(x) = \min(AD + DB) = AB$ .



Bunda  $f(x) = \sqrt{x^2 - 5x\sqrt{2} + 25} + \sqrt{x^2 - 12x\sqrt{2} + 144}$   $x$  ning qiymatini topish uchun 2-misoldagidek ish yuritamiz. Agar tenglama ildizlarga ega bo'lsa, ular musbat bo'lishni aniqlaymiz. ( $x \leq 0$  da tenglama chap qismining qiymati 12 dan kichik emas).

$$S_{\triangle ABC} = \frac{1}{2} \cdot 5 \cdot 12 = 30.$$

$$S_{\triangle ACD} = \frac{1}{2} \cdot 5 \cdot x \cdot \sin 45^\circ = \frac{5x\sqrt{2}}{4}.$$

$$S_{\triangle BCD} = \frac{1}{2} \cdot 12 \cdot x \cdot \sin 45^\circ = 3x\sqrt{2}.$$

Demak,  $30 = \frac{5x\sqrt{2}}{4} + 3x\sqrt{2}; \quad x = \frac{60\sqrt{2}}{17}.$

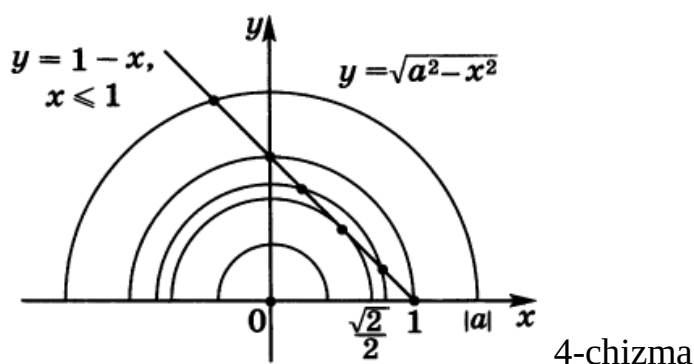
Javob:

$$\frac{60\sqrt{2}}{17}.$$

4.  $a$  parametrning har bir qiymatida  $\sqrt{a^2 - tg^2 z} = 1 - tgz$  tenglamada  $tgz$  nechta har xil qiymatlar qabul qiladi.

Yechish. Bu oddiy tenglama emas. Uni an'anaviy usulda yechish oson emas, agar  $tgz$  ni  $x$  bilan almashtirsak, uni geometrik usulda yechish oson bo'ladi.

4-chizmada  $y = \sqrt{a^2 - x^2}$  (markazi koordinatalar boshida, radiusi  $|a|$  bo'lgan yuqori yarim tekislikda joylashgan yarim aylana) va  $y = 1 - x$  ( $x \leq 1$ ) (boshi  $(1;0)$  nuqtada  $(0,1)$  nuqtadan o'tuvchi nur) funksiyalar grafiklari tasvirlangan.



Demak,  $\sqrt{a^2 - x^2} = 1 - x$

$|a| < \frac{\sqrt{2}}{2}$  da ildiz yo'q.

$|a| = \frac{\sqrt{2}}{2}$  va  $|a| > 1$  da 1 ta ildizga ega.

$\frac{\sqrt{2}}{2} < |a| \leq 1$  da 2 ta ildizga ega.

Demak,  $tgz$ :

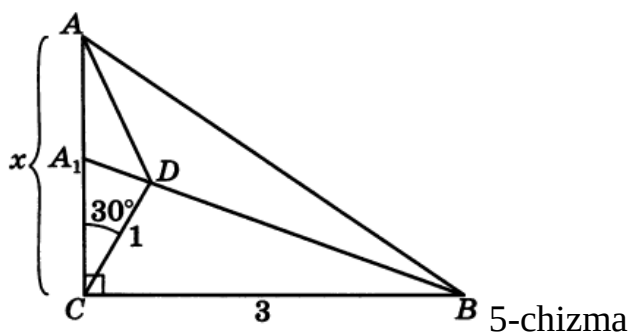
$|a| = \frac{\sqrt{2}}{2}$  da bitta qiymat qabul qiladi. ( $-\frac{\sqrt{2}}{2}$  da bitta son bo'lsa,  $\frac{\sqrt{2}}{2}$  da boshqa son).

$|a| > 1$  da bitta qiymat qabul qiladi.

$\frac{\sqrt{2}}{2} < |a| \leq 1$  da 2 ta har xil qiymat qabul qiladi.

5.  $f(x) = \sqrt{x^2 + 9} - \sqrt{x^2 - x\sqrt{3} + 1}$  funksiyaning eng katta qiymatini toping.

Yechish.  $ABC$  uchburchakni qaraymiz. Bunda  $\angle ACB = 90^\circ$ ,  $\angle ACD = 30^\circ$ ,  $AC = x$ ,  $BC = 3$ ,  $CD = 1$  va  $D$  nuqta  $ABC$  uchburchak ichida yotadi. (5-chizma)



$ABC$  uchburchakdan Pifagor teoremasiga ko'ra  $AB = \sqrt{x^2 + 9}$ .

$\triangle ACD$  dan kosinuslar teoremasiga ko'ra  $AD = \sqrt{x^2 - x\sqrt{3} + 1}$ .

$$\max f(x) = \max(AB - AD) = A_1B - A_1D = DB,$$

bunda  $A_1 \in AC$  (agar  $D \in AB$ ).

$\triangle BCD$  dan kosinuslar teoremasiga ko'ra

$$DB = \sqrt{1^2 + 3^2 - 2 \cdot 1 \cdot 3 \cos 60^\circ} = \sqrt{7}.$$

Javob:  $\sqrt{7}$ .

6.  $f(x) = x\sqrt{2} + \sqrt{2x^2 + 2x + 1} + \sqrt{2x^2 - 14x + 25} + \sqrt{2x^2 - 26x + 89}$  funksiyaning eng kichik qiymatini toping.

Yechish.  $\vec{a}(x; x)$ ,  $\vec{b}(4 - x; 3 - x)$ ,  $\vec{c}(x + 1; x)$ ,  $\vec{d}(5 - x; 8 - x)$  vektorlar va ularning modullarini qaraymiz.

$$|\vec{a}| = x\sqrt{2}.$$

$$|\vec{b}| = \sqrt{2x^2 - 14x + 25}.$$

$$|\vec{c}| = \sqrt{2x^2 + 2x + 1}.$$

$$|\vec{d}| = \sqrt{2x^2 - 26x + 89}.$$

$$f(x) = |\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}|.$$

$$|\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}| \geq |\vec{a} + \vec{b} + \vec{c} + \vec{d}| \text{ bo'lgani uchun}$$

$$\min f(x) = |\vec{a} + \vec{b} + \vec{c} + \vec{d}|.$$

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\vec{10}; \vec{11})$$

$$|\vec{a} + \vec{b} + \vec{c} + \vec{d}| = \sqrt{221}.$$

$$\text{Demak, } \min f(x) = \sqrt{221}.$$

$$\text{Javob: } \sqrt{221}.$$

7.  $f(x) = \sqrt{2x^2 - 4x + 2} + \sqrt{2x^2} + \sqrt{2x^2 - 2x + 2}$  funksiyaning eng kichik qiymatini toping.

$$\text{Yechish. 4 ta } O(0;0), A(1;1), B\left(\frac{1-\sqrt{3}}{2}; \frac{1+\sqrt{3}}{2}\right), M(x;x) \text{ va } OM, AM \text{ va } BM$$

$$\text{masofalarni qaraymiz. } OM = \sqrt{2x^2}, \quad AM = \sqrt{2x^2 - 4x + 2}, \quad BM = \sqrt{2x^2 - 2x + 2}.$$

$$f(x) = OM + AM + BM.$$

$ABO$  uchburchakning tomonlarini hisoblash qiyin emas.

$AO = BO = AB = \sqrt{2}$ . Demak,  $\triangle ABO$  teng tomonli

$$\min(AM + OM + BM) = 3R,$$

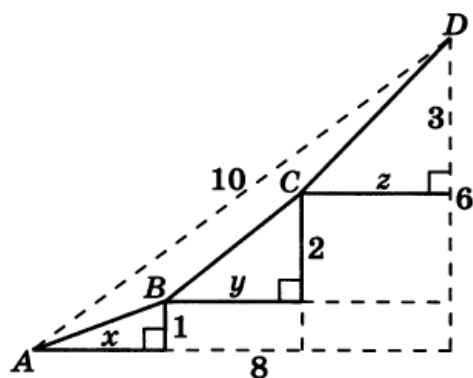
$R$ -teng tomonli uchburchakka tashqi chizilgan aylana radiusi.

$$\sqrt{2} = R\sqrt{3} \text{ bo'lgani uchun } \min f(x) = \sqrt{6}.$$

Javob:  $\sqrt{6}$ .

8.  $f(x; y; z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 4} + \sqrt{z^2 + 9}$  va  $x + y + z = 8$  bo'lsa,  $f(x; y; z)$  funksiyaning kichik qiymatini toping.

Yechish.



6-chizma

6-chizmaga ko'ra  $ABCD$  egri chiziq uzunligi 10 dan kichik emas, u holda  $x + y + z = 8$  da  $f(x; y; z) = 10$ .

Javob: 10.



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