KASR TARTIBLI FAZOVIY IKKI O‘LCHOVLI BIR JINSLI BO‘LMAGAN TESKARI MASALA

Sag‘dullayeva Marjona Uchqun qizi

Annotatsiya: Ushbu tezisda kasr tartibli ikki o‘lchovli bir jinsli bo‘lmagan teskari masala va ularning yechimlari metodlari chuqur tahlil qilinadi. Kasr tartibli integro-differensial tenglamalar ko‘p sohalarda murakkab jarayonlarni modellashda ko‘p qo‘llaniladi. Kasr operatorlari esa kasr tartibli differensial tenglamalarga yangi nazariy yondashuvlarni olib kirib, ularning sifatli xususiyatlarini o‘rganishga xizmat qiladi.

Kalit so‘zlar: Hilfer kasr operatori, kasr tartibli integro-differensial tenglamalar, siquvchi akslantirish metodi,matematik modellashtirish.

Kasr tartibli integro-differensial tenglamalar so‘nggi yillarda matematikaning faol rivojlanayotgan yo‘nalishlaridan hisoblanadi. Ular ko‘p jihatdan oddiy differensial tenglamalardan farq qilib, o‘zida kasr tartibli hosilalar va integral operatorlarini mujassamlashtiradi. Ayniqsa Hilfer tomonidan taqdim etilgan kasr tartibli operatorlar, turli xil operatorlar orasida iqtisodiy, mexanik va biomeditsina sohalaridagi modellarni yanada qulay va aniq ifodalashga imkon beradi. Hilfer operatori Rieman-Liuvill va Kaputo operatorlari orasidagi umumlashma hisoblanib, uning xossalari tenglamalarda yechimlarning sifatini ta’minlashda katta rol o‘ynaydi

Kasr tartibli integro-differensial tenglamalar sistema dinamikasidagi xotira ta’siri, vaqtga bog‘liq o‘zgarishlarning noan’anaviy o‘lchamlarini tushuntirishda qulay vositadir. Shu sababli ularga iqtisodiyot, fizik jarayonlar, biologik tizimlar va boshqalarda keng qo‘llaniladi. [1].

Kasr tartibli differensial tenglamalarda noma’lum funksiyaning hosilasi butun sonli emas, balki ixtiyoriy haqiqiy tartibli bo‘lishi mumkin. Bunday tenglamalarni o‘rganishda Riman–Liuvill, Kaputo va Hilfer kasr tartibli hosilalaridan foydalaniladi. Kasr tartibli differensial tenglamalarni yechish

an’anaviy differensial tenglamalarga qaraganda murakkabroq va ko‘pincha analitik yechim olish qiyin bo‘ladi. Shu sababli zamonaviy metodlar va usullar qo‘llaniladi. Yechim olishning asosiy yo‘nalishlaridan biri quyidagi hisonlanadi: [2]

- Siquvchi akslantirish metodi: Ushbu usul yechimning mavjudligi va yagonaligini isbotlash uchun qo‘llaniladi.Unga ko‘ra operatorning yagona qo‘zg‘almas nuqtasi mavjud va bu nuqta dastlabki kasr tartibli differensial tenglamaning yagona yechimi hisoblanadi[2].

Kasr tartibli differensial tenglamalar nafaqat nazariy izlanishlarda, balki ko‘plab amaliy sohalarda ham keng qo‘llaniladi. Ularning xotira va esdan chiqarish effektlarini hisobga olish qobiliyati, tabiiy jarayonlarda mavjud noan’anaviy dinamikani yaxshiroq tavsiflash imkonini beradi. Misol uchun:

- Fizika: Kasr tartibli differensial tenglamalar to‘qimalar va materiallarning mexanik xatti-harakatlarini tushuntirishda, ayniqsa materiallarning deformatsiyasini o‘rganishda qo‘llaniladi.

Hilfer turidagi kasr operatori, psevdoparabolik operatorning kvadrati hamda berilgan kesmaning oxirgi nuqtasida qayta aniqlanuvchi funksiya ishtirok etgan chiziqli bo‘lmagan differensial tenglama uchun teskari chegaraviy qiymat masalasi o‘rganiladi. Ushbu teskari chegaraviy masalaning yechimining mavjudligi va yagonaligi masalalari tahlil qilinadi.

Berilgan $\Omega \equiv (0,T)\times(0,l)\times(0,l)$ sohada quyidagi tenglamani ko‘rib chiqamiz.

$$\left[D_{0t}^{\alpha,\gamma} - D_{0t}^{\alpha,\gamma}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) - \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]^2 U(t,x,y) = F(t,x,y) \qquad (1)$$

chegaraviy shartlar bilan

$$\lim_{t\to+0} I_{0t}^{1-\gamma} u(t,x,y) = \varphi_1(x,y), u(T,x,y) = \varphi_2(x,y), \qquad (2)$$

$$u(t,0,y) = u(t,l,y) = u_{xx}(t,0,y) = u_{xx}(t,l,y) = u_{yy}(t,0,y) = u_{yy}(t,l,y)$$

$$=$$

$$= u(t,x,0) = u(t,x,l) = u_{xx}(t,x,0) = u_{xx}(t,x,l) = u_{yy}(t,x,0) =$$

$$u_{yy}(t,x,l)=0, \quad (3.1.3)$$

o‘rganamiz.

bu yerda$\varphi_\kappa(x,y)\in C(\Omega_l^2)$, $0<\alpha\le\gamma\le 1$,

$$F(t,x,y)\in C(\Omega_T\times\Omega_l^2),\ \kappa=1,2,$$

$$\varphi_\kappa(0,y)=\varphi_\kappa(l,y)=\varphi''_{xx,\kappa}(0,y)=\varphi''_{xx,\kappa}(l,y)=\varphi''_{yy,\kappa}(0,y)=\varphi''_{yy,\kappa}(l,y)=$$
$$=\varphi_\kappa(x,0)=\varphi_\kappa(x,l)=\varphi''_{xx,\kappa}(x,0)=\varphi''_{xx,\kappa}(x,l)=\varphi''_{yy,\kappa}(x,0)=\varphi''_{yy,\kappa}(x,l)$$
$$=0,$$

$$F(t,0,y)=F(t,l,y)=F_{xx}(t,0,y)=F_{xx}(t,l,y)=F_{yy}(t,0,y)=F_{yy}(t,l,y)=$$
$$=F(t,x,0)=F(t,x,l)=F_{xx}(t,x,0)=F_{xx}(t,x,l)=F_{yy}(t,x,0)=F_{yy}(t,x,l)$$
$$=0,$$

$$\Omega_T\equiv[0,T],\ \Omega_l^2=\Omega_l\times\Omega_l,\ \Omega_l\equiv[0,l],\ 0<T<\infty,\ 0<l<\infty.$$

1-masala. (1) Kasr differensial tenglamaning (2) ikki nuqtali chegaraviy shartlari hamda (3) Dirixle tipidagi shartlarni qanoatlantiruvchi va berilgan funksiyalar sinfiga tegishli bo‘lgan $U(t,x,y)$ funksiyani yechish talab qilinadi.

$$t^2D^{\alpha,\gamma}U\in C(\overline{\Omega}),\ t^2D^{\alpha,\gamma}U_{xxxx}\in C(\Omega),\ t^2U_{xxxx}\in C(\Omega),$$
$$t^2D^{\alpha,\gamma}U_{yyyy}\in C(\Omega),\ t^2U_{yyyy}\in C(\Omega), \quad (4)$$

bu yerda $\bar{\Omega}\equiv\Omega_T\times\Omega_l^2$.

Keyingi tahlil uchun quyidagi farazlar qabul qilinadi:

$$U(t,x,y)=\sum_{n,m=1}^{\infty}u_{n,m}(t)\cdot\vartheta_{n,m}(x,y), \quad (5)$$

va

$$F(t,x,y)=\sum_{n,m=1}^{\infty}f_{n,m}(t)\cdot\vartheta_{n,m}(x,y),(t,x,y)\in\Omega, \quad (6)$$

bu yerda

$$u_{n,m}(t)=\int_0^l\int_0^l U(t,\eta,\xi)\vartheta_{n,m}(\eta,\xi)d\eta d\xi\,,f_{n,m}(t)=$$
$$\int_0^l\int_0^l F(t,\eta,\xi)\vartheta_{n,m}(\eta,\xi)d\eta d\xi\,, \quad (7)$$

$$\vartheta_{n,m}(x,y)=\frac{2}{l}sin\sqrt{\lambda_n}\,x\,sin\sqrt{\lambda_m}\,y, \quad (8)$$

$$\lambda_n=\left(\frac{\pi n}{l}\right)^2,\lambda_m=\left(\frac{\pi m}{l}\right)^2,n,m=1,2,\ldots$$

$\{\vartheta_{n,m}(x,y)\}_{n,m=1}^{\infty}$ funksiyalar sistemasi $L_2(\Omega_l^2)$ fazoda ortonormal

funksiyalarning to'liq tizimini hosil qiladi va chegara shartlarini qanoatlantiradi.

Silliqlik sharti 1. Aytaylik $\varphi_k(x,y) \in C^{5,5}(\Omega_l^2)$ va $F(t,x,y) \in C^{0,2,2}(\Omega)$ bo‘lsin.

1-teorema. Faraz qilaylik silliqlik sharti bajarilsin. U holda (1)–(4) to‘g‘ri masala Furye qatori ko‘rinishida yagona yechimga ega bo‘ladi.

Xulosa

Kasr tartibli aynigan integro-differensial tenglamalar hamda psevdoparabolik operator kvadrati qatnashgan ikki o‘lchovli bir jinsli bo‘lmagan differensial tenglamalar uchun teskari chegaraviy masalalar o‘rganildi. Hilfer operatori Riman–Liuvill va Kaputo turidagi kasr tartibli hosilalarni umumlashtiruvchi operator sifatida qaraladi. Shu sababli uning yordamida turli boshlang‘ich, chegaraviy va chiziqli bo‘lmagaan masalalarni yagona yondashuv asosida tadqiq qilish mumkin.

Foydalanilgan adabiyotlar

1. R. R. Ashurov, Yu. E. Fayziev, Inverse problem for finding the order of the fractional derivative in the wave equation, Math. Notes 110 (6), 842-852 (2021).

2. S. Z. Djamalov and R. R. Ashurov, On one linear inverse problem for multidimensional equation of the mixed type of the first kind and of the second order, Russian Math. (Iz. VUZ) 63 (6), 8-18 (2019).

3. V. G. Romanov, Phaseless inverse problems that use wave interference, Siberian Math. Journ. 59 (3), 494-504 (2018).

4. T.K.Yuldashev, M. T. Kosmakova, M. U. Sagdullaeva, Inverse Problem for a Nonlinear Pseudiparabolic Differential Equation with Final Condition and Gerasimov-Caputo Operator, Journal of Osh University :Differential Equations” 2026

5. Qodirkulov B.J., Kayumova G.A. Hilfer operatori qatnashgan ikkinchi tartibli aralash tipdagi tenglama uchun nolokal masala. — Matematika instituti ilmiy axborotnomasi, 2021.

6. Jalilov M.A. Kasr tartibli differensial operator qatnashgan aralash tipdagi tenglamalar uchun masalalar. — Matematika instituti ilmiy axborotnomasi, 2019.